



Sample-Based Matroid Prophet Inequalities

HU FU, Shanghai University of Finance and Economics, China

PINYAN LU, Shanghai University of Finance and Economics, China

ZHIHAO GAVIN TANG, Shanghai University of Finance and Economics, China

HONGXUN WU, UC Berkeley, USA

JINZHAO WU, Yale University, USA

QIANFAN ZHANG, Princeton University, USA

The classical prophet inequalities problem introduced by Krengel and Sucheston [1977, 1978] assumed complete knowledge of distributions. However, such an assumption may be unrealistic both in practice and for some applications.

We study matroid prophet inequalities when distributions are unknown and accessible only through samples. While single-sample prophet inequalities for special matroids are known, no constant-factor competitive algorithm with even a sublinear number of samples was known for general matroids. Adding more to the stake, the single-sample version of the question for general matroids has close (two-way) connections with the long-standing matroid secretary conjecture.

In this work, we give a $(1/4 - \epsilon)$ -competitive matroid prophet inequality with only $O_\epsilon(\text{poly } \log n)$ samples. Our algorithm consists of two parts: (i) a novel *quantile-based* reduction from matroid prophet inequalities to online contention resolution schemes (OCRSs) with $O_\epsilon(\log n)$ samples, and (ii) a $(1/4 - \epsilon)$ -selectable matroid OCRS with $O_\epsilon(\text{poly } \log n)$ samples which carefully addresses an adaptivity challenge.

A full version of this paper can be found at <https://arxiv.org/abs/2406.12799>.

CCS Concepts: • **Theory of computation** → **Online algorithms**.

Additional Key Words and Phrases: prophet inequalities, online contention resolution schemes

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Authors' Contact Information: Hu Fu, fuhu@mail.shufe.edu.cn, Key Laboratory of Interdisciplinary Research of Computation and Economics and Institute for Theoretical Computer Science, Shanghai University of Finance and Economics, China; Pinyan Lu, lu.pinyan@mail.shufe.edu.cn, Key Laboratory of Interdisciplinary Research of Computation and Economics and Institute for Theoretical Computer Science, Shanghai University of Finance and Economics, China; Zhihao Gavin Tang, tang.zhihao@mail.shufe.edu.cn, Key Laboratory of Interdisciplinary Research of Computation and Economics and Institute for Theoretical Computer Science, Shanghai University of Finance and Economics, China; Hongxun Wu, wuhx@berkeley.edu, UC Berkeley, USA; Jinzhao Wu, jinzhaowu@yale.edu, Yale University, USA; Qianfan Zhang, qianfan@princeton.edu, Princeton University, USA.

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